

# Real options for flexible power generation modeling

Michael Siclari and Giuseppe Castellacci of OpenLink's Risk and Analytics Group extend their real option models for valuation of power generation valuation to capture all stochastic factors and not just fuel costs

As more and more environmental and regulatory issues affect the costs of power generation, it is becoming increasingly important to have tools available that can estimate the impact of these factors on the profitability of power generation. Now factors other than fuel, and fixed and variable costs can have a significant impact on a generation asset valuation. A case in point is that of emissions. Emission policies have led to a market for emission permits to facilitate compliance. Power generation asset holders may have generators that use a mix of fuels, such as coal, oil or natural gas. Gas-fired plants typically have lower carbon emissions than coal-fired ones. The generation asset managers now have to evaluate the cost as well as the emission characteristics of different fuels in conjunction with the operating costs of the plant. If the market price of emission permits rise, the manager may opt to build or acquire a gas-fired generator to reduce costs.

In [CSRL04] (see references), the authors introduced spread switching options to value the power generation of a co-firing generator or a pair of generators that can run on different fuels. This article extends that option to a *basket spread switching option*,

where the cost of power generation can be affected by any number of stochastic factors, such as the price of fuel and the cost of emission permits.

A complete suite of option tools are now available for the modeling and rapid valuation of one or more power plants or co-firing generators. These, in order of complexity and accuracy, include:

Single plant, single fuel:

- spark spread (single factor cost);
- spread (single factor cost with fixed cost); and
- basket spread (multiple cost factors with fixed cost)

Co-firing or multiple plants:

- spread (fuel) switching (two cost factors or fuels with fixed cost); and
- basket spread switching (multiple cost factors with fixed cost)

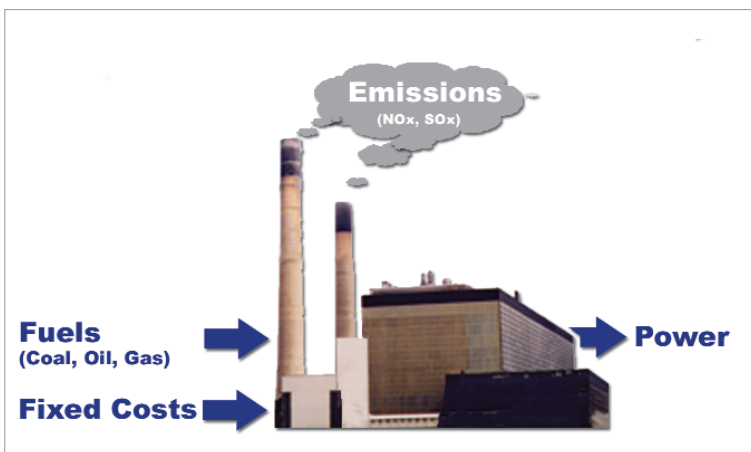
The remainder of this paper presents approximate pricing models for the valuation of European spread switching options on baskets. The payoff can be thought of as the spread between the price of power and the minimum of two baskets of stochastic cost factors, such as the price of fuel, emission costs, or any other factors affecting the cost of power generation. The pricing models are semi-analytical and hybrid in nature, for they first reduce the basket switching option to a spread switching option by approximating the baskets with log-normal variables as in [CS03], then they price the resulting spread switching option with the fast methods of [CSRL04].

The payoff of a basket spread switching option can be defined as:

$$V_{Call} = \max \{w_0 P_0 - \min [B_1, B_2] - X, 0\} \quad (1)$$

where

$$B_1 = \sum_{j=1}^{N_1} w_{1j} P_j \quad B_2 = \sum_{j=1}^{N_2} w_{2j} P_j$$





the  $P_i$ 's are factors (prices), the  $w_i$ 's are weights or conversion factors, and  $B_1$  and  $B_2$  are two baskets each comprising  $N_1$  and  $N_2$  factors, respectively (e.g., fuels, emissions, etc).

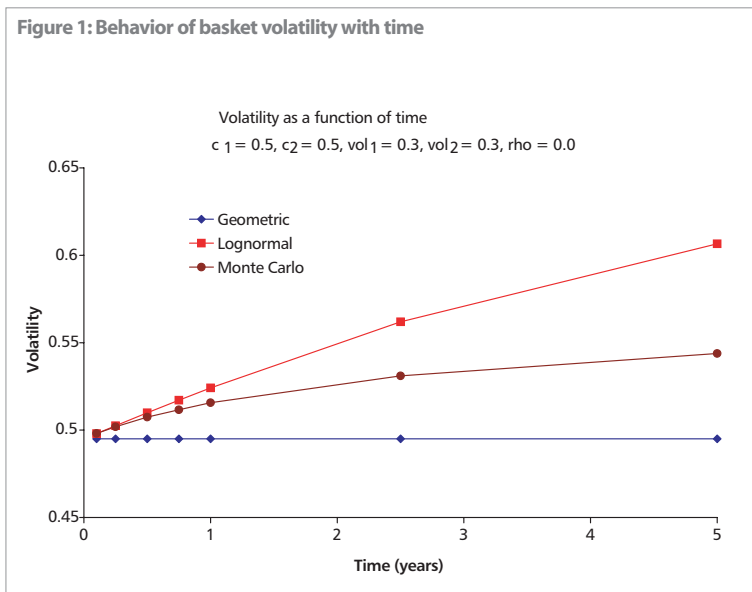
### Formulation of the hybrid-pricing model

The hybrid model consists of two stages. In the first, we approximate each basket with one lognormal variable (see [CS03]). There are two simple ways to achieve this: one approximates with geometric averages (after the work of Vorst [Vo95] and Gentle [Ge95]); the other is based on matching the moments of the approximating variables to the baskets moments (see Levy [Le92]). The most important technicality of this part of the models is the calculation of the appropriate variance of the logarithm of the approximating variables, as well as the correlations among them. Using these approximations, the initial payoff can be reduced to a spread-switching option's (the payoff is the same as (1) with single factors instead of baskets). Now we can apply the fast valuation techniques of [CSRL04]. Those models converge to the exact value (under Black-Scholes or similar log-normal assumptions). The model at hand, however, is always approximate because of the approximation of the first stage.

Next, we illustrate the calculation of the variances and correlation for both lognormal approximations of the first modeling stage. According to a common practice, one renormalizes the weights of the  $i$ -th basket ( $i=1, 2$ ) as follows:

$$c_{ik} = \frac{w_{ik} F_{ik}}{\sum_{j=1}^{N_i} w_{ij} F_{ij}}$$

where  $F_{ik}$  stands for the forward price (or expectation) of the  $k$ -th factor of basket  $B_i$ .



### Geometric averaging method

When one uses the geometric average as the approximating log-normal variable, the variance of basket  $B_i$  is equal to:

$$v_{B_i} = \sigma_{B_i}^2 T_{exp} = \sum_{j=1}^{N_i} \sum_{k=1}^{N_i} c_j c_k \rho_{jk} \sigma_j \sigma_k \min(T_j, T_k)$$

where  $T_{exp}$  represents the time to expiration,  $\rho_{jk}$  is the correlation between the  $j$ -th and  $k$ -th factor return,  $\sigma_{B_i}$  the (equivalent) basket volatility, and the  $T_j$ 's are the time to expiration of each asset. The correlations between the two baskets and between the long factor (asset 0) and each basket is:

$$\rho_{B_1, B_2} = \frac{\sum_{j=1}^{N_1} \sum_{k=1}^{N_2} c_{1j} c_{2k} \rho_{jk}^{B_1, B_2} \sigma_j^{B_1} \sigma_k^{B_2} \min(T_j^{B_1}, T_k^{B_2})}{\sqrt{v_{B_1} v_{B_2}}}$$

$$\rho_{P_0, B_i} = \frac{\sum_{j=1}^{N_i} c_{ij} \rho_{0,j} \sigma_0 \sigma_j \min(T_0, T_j^{B_i})}{\sigma_0 \sigma_{B_i} \sqrt{T_0 T_{exp}}}$$

### Lognormal averaging method

The lognormal approximation for the basket values leads to the following formulas for the variances and correlations:

$$v_{B_i} = \sigma_{B_i}^2 T_{exp} = \ln \left( \sum_{j=1}^{N_i} \sum_{k=1}^{N_i} c_{ij} c_{ik} e^{\rho_{jk} \sigma_j \sigma_k \min(T_j, T_k)} \right)$$

$$\rho_{B_1, B_2} = \frac{\ln \left( \sum_{j=1}^{N_1} \sum_{k=1}^{N_2} c_{1j} c_{2k} e^{\rho_{jk}^{B_1, B_2} \sigma_j^{B_1} \sigma_k^{B_2} \min(T_j^{B_1}, T_k^{B_2})} \right)}{\sqrt{v_{B_1} v_{B_2}}}$$

$$\rho_{P_0, B_i} = \frac{\ln \left( \sum_{j=1}^{N_i} c_{ij} e^{\rho_{0,j} \sigma_0 \sigma_j \min(T_0, T_j^{B_i})} \right)}{\sigma_0 \sigma_{B_i} \sqrt{T_0 T_{exp}}}$$

### Basket variance and volatility behavior

To test the behavior of the basket volatility, consider the simple case in which both baskets comprise two factors and there is a single expiration time ( $T$ ) for all assets.

The volatility of the basket assets is fairly high (70%), which should stress the accuracy of the approximation. The weights are equal and the correlation is zero. Figure 1 shows a plot of the volatility as a function of time.

One million trial Monte Carlo simulations were carried out to assess the behavior of each model's basket volatility. Figure 1 compares the simulated (nearly 'true') volatility with those coming from the approximate valuation models. One can see that the former lies between the latter. Notice also that the volatility com-

ing from the geometric average model is constant in time, whereas the others grow. For shorter times to expiration, the lognormal model appears to be in better agreement with the simulated results. Beyond one year, the lognormal model overestimates the volatility obtained from the Monte Carlo simulation.

**Model validation with Monte Carlo simulations**

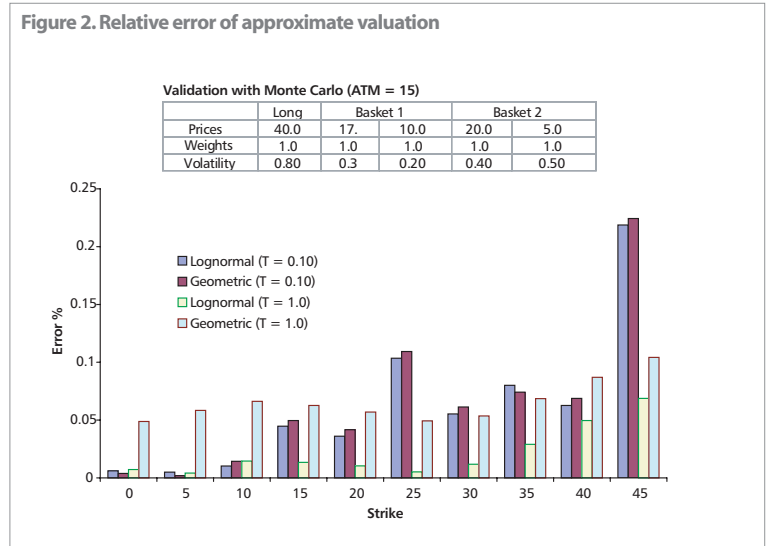
To study the accuracy of these models, we selected a sample set of parameters and tested the model as the moneyness changed. The interest rate was 1% and the dividend rate was equal to the former (futures or forward prices in a Black-Scholes framework). Each basket comprises two factors for a total of five underlying factors. The correlation matrix was constant with off diagonal values of 0.5. We validated the approximate semi-analytical models by comparing them to Monte Carlo simulations with antithetic variance reduction each comprising one million trials.

Figure 2 shows the percentage errors of these values in comparison to the Monte Carlo values. All moneyness values are predicted accurately (to less than 0.25%) by both the geometric and the lognormal models even for values that are deeply out of the money (OTM).

**Application of spread switching options to power plant valuation**

As mentioned above, the most immediate application of basket spread switching options is for the valuation of power generation as a real option. Figure 3 shows an example of the application of spread and spread switching options to an hourly option valuation for a power generation monthly output based on an hourly spot forecast for power and two different fuels. The spread switch-

Figure 2. Relative error of approximate valuation

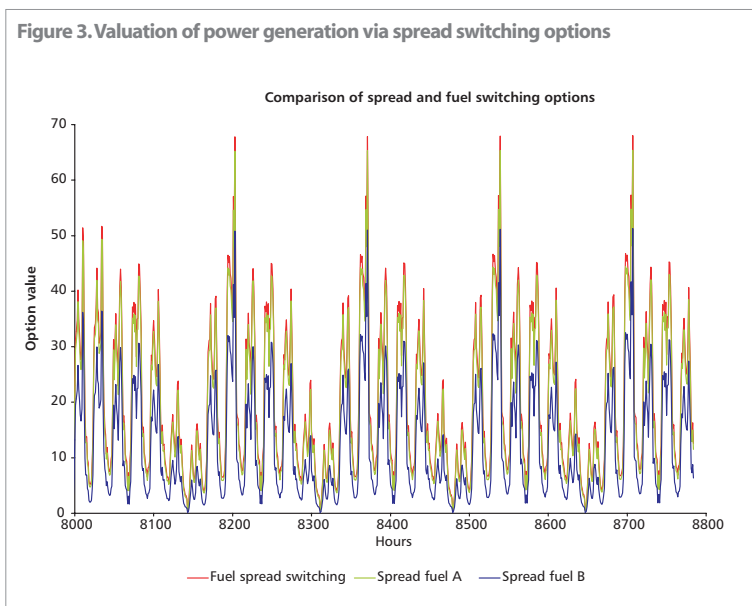


ing option captures the additional value-added flexibility of switching generation from two different fuels.

**Summary**

This article has introduced basket spread switching options, a new kind of spread option that models the profit from the sale of one asset with the choice of the cheaper of two baskets of stochastic cost factors and one fixed cost. We presented two approximate valuation models for this class of options. Both models are computationally efficient and produce relative errors of less than 1% when compared to a one-million trial Monte Carlo simulation for a selected example. These models have important applications to the real option valuation of power generation assets and related risk management tasks, such as the estimation of earnings at risk. In particular, they capture not only the additional optionality coming from fuel switching, but also the presence of multiple stochastic cost factors. <sup>ERI</sup>

Figure 3. Valuation of power generation via spread switching options



**References**

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